## A New Automatic Frequency Regulation System*

### Introduction

A considerable number of frequency regulation systems for voltage tunable sources such as klystrons are presently available. The most prominent of these is the Pound Regulator[1] which provides an excellent stable source. Perhaps the only disadvantages of the Pound system are the initial cost and the rather critical adjustments necessary when starting up.

The system described here was designed for stabilization of a VA-96 ($K$-band) klystron, but is readily applicable to any other voltage-tunable microwave source. In essence, the error signal is derived from a phase comparison of two signals. One is a driving signal of 100 kilocycles per second which is also used to modulate a reference cavity (in size), and the second signal is the rectified microwave signal reflected from the reference cavity. A block diagram of the system is shown in Fig. 1.

The advantages of this system are that it is economical in equipment, and it is easy to align and adjust. Furthermore, the output is monochromatic and isolated from the stabilization loop. Additional isolation may be secured, if desired, by use of a ferrite isolator.

### Cavity Modulation

Several means of modulating the cavity dimensions were considered. Magnetostriction driving, mechanical oscillations, and piezoelectric crystals are all feasible. The choice may be made on the basis of obtaining an error signal from the phase detector in a time much shorter than the reciprocal of the highest harmonic of the noise. For example, in the case of klystrons operating from rectified ac, the highest noticeable harmonic of the ripple would likely be less than 1000 cycles per second; hence the system response should be faster than one millisecond.

A mechanical modulation system using a cam and motor or solenoid driven end plate for the cavity is not easily driven at frequencies in excess of a few hundred cycles per second. For high frequencies it is much more feasible to utilize either magnetostrictive or piezoelectric devices. Using such mechanisms, frequencies of 100 kilocycles or more are obtainable which permit sufficiently fast response times even after the integration of a phase detector.

### Experimental Results

Experimentally, it was discovered that piezoelectric modulation was more easily adaptable to a $K$-band cavity than magnetostriction modulation. Using a 100-kc 50-volt peak-to-peak signal applied to a Rochelle salt crystal, the resonant frequency of the cavity could be varied from about 22.2 kmc to 24.2 kmc. The cavity was

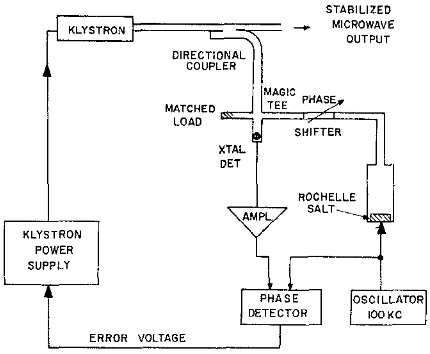


Fig. 1—Block diagram of the modulated cavity frequency stabilization scheme.

constructed with a foil-plated Rochelle salt crystal as one end plate. The crystal was obtained from a headphone and aluminum foil was cemented to the crystal face. It is likely that a quartz crystal would permit operation at higher frequencies if desired.

Frequency stability measurements have thus far been confined to comparing the frequency drift with and without the feedback loop. It appears that the system is stable to better than one megacycle, and probably is stable to within 50 kc. However, we have not experimentally measured the deviation precisely since we have no available stable source for comparison. Future experiments will utilize this source for paramagnetic resonance experiments and will permit a more exact determination of frequency drift. The theoretical calculation of stability involves the gain of the feedback loop. For the present setup, the stability should theoretically be within 50 kc.

### Acknowledgment

I am indebted to G. D. Ewing for carrying out the experimental measurements, and to Prof. A. M. Portis for an interesting discussion of other stabilization methods.

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[1] R. V. Pound, "Frequency stabilization of microwave oscillators," Proc. IRE, vol. 35, pp. 1405–1415; December, 1947.

## On Some Problems in Designing Microwave Faraday-Rotation Devices*

During recent years a substantial effort has been made to obtain special ferrites for microwave applications. The authors' opinion is that in many cases this effort was not supported by real necessity and that many existing commercial materials may reveal surprising microwave properties after a persistent and careful preparation of working conditions.

An investigation of the microwave behavior of a Ferroxcube B5-type ferrite was carried out in this laboratory and the material was not found to be less attractive than many ferrites designed especially for the $X$ band.

Faraday rotation in cylindrical structures with longitudinal magnetization has been studied in detail. Substantial experience was gained in measuring techniques and soon it became apparent that many data on Ferroxcube B5 quoted in the literature are erroneous. Causes of these errors are ascribed to higher mode effects which seriously affect the measurements if only ferrite rod diameter is greater than some critical value. Critical diameters experimentally determined were found to be in good agreement with theoretical calculations by Waldron.[1]

The optimum sample geometry was determined, and for this geometry Ferroxcube B5 figure of merit values were obtained up to 250°/db, with losses not greater than 0.1 db/inch. Broad-band isolators and circulators were next built and their total forward loss was found to be less than 0.35 db, reverse loss being greater than 40 db over the 8600–10,000-mc band, and greater than 60 db at the middle 300 mc.

Two basic systems were used throughout this work. The first one was used to measure the figure of merit as a function of ferrite rod diameter. A special instrument ("Faraday Rotating Meter") working on principles given by Hogan was designed for this purpose. The angle of Faraday rotation was measured with an accuracy of $\pm 0.05°$ and the corresponding losses with an accuracy of $\pm 0.05$ db. Several facilities were provided to enable quick measurements of a large number of samples.

All other measurements were carried out with a fixed angle of Faraday rotation equal to 45°, as this angle is the most important for practical applications. A schematic diagram of an experimental arrangement designed for this purpose is given in Fig. 1.

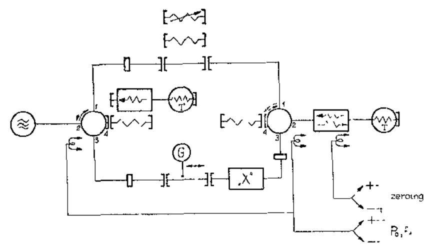

Fig. 1—Universal set-up.

In this arrangement, the two circulators, magnetized from a common dc power supply and controlled by a common switch, are directing the power flow from the generator to the power meter through one of the two transmission paths available. In one position of the switch, the power enters the power meter through the reference path; in the other position, it travels through the device under test before entering the thermistor mount. A comparison of two deflections of the power indicator, corresponding to the two positions of the switch, gives the attenu-

* Received by the PGMTT, October 6, 1959.

[1] R. A. Waldron, "Electromagnetic wave propagation in cylindrical waveguides containing gyromagnetic media," J. Brit. IRE, vol. 18, pp. 733–746; December, 1958.

ation of the device in question. The substitution method can also be used. A switched ferrite isolator is inserted before the thermistor mount, thus separating the measurement path from reflections and also offering a possibility of zeroing the thermistor bridge.

The VSWR measurement can be made by means of conventional slotted line techniques or by connecting a crystal mount to the fourth port of the first circulator and maintaining a constant power output from the generator. Under such conditions, any indicator connected with the crystal can be scaled in terms of the reflection coefficient. This simple circulator reflectometer was found to be particularly efficient when a large number of measurements were carried out at a fixed frequency; the same restriction was valid for the whole setup given in Fig. 1. In the final stage of measurements, when maximum accuracy was needed, conventional setups were preferred.

The measured ferrite sample was inserted into IK-1M isolator.[2] A schematic cross section thorugh the main part of this device is given in Fig. 2. Iron flanges and tubing are used for focusing the magnetic field and for forming a special axial distribution of this field, which is flat at the middle part and rapidly decreases to the ends of the circular guide (Fig. 3). In some applications (circulators) the ferrite was placed off the center of symmetry of this distribution (but at the center of the guide) and thus additional tuning was obtained.

The ferrite rod was supported in circular guide by polyfoam or, in final stage of measurements, by polystyrene screws inside a polystyrene sleeve, closely fitting the guide walls.

All measured samples had the shape of a cylinder with 1-inch-long tapers at both ends. Ferrite pieces 2 inches long were ground to the desired length and diameter, the matching taper was made at one end, and then two such pieces were glued together to obtain the final shape; this procedure reduced the influence of the individual properties of the samples. During the measurements only the length and the diameter of the cylindrical part varied; the length of matching tapers was kept constant. Generous water cooling was provided during grinding.

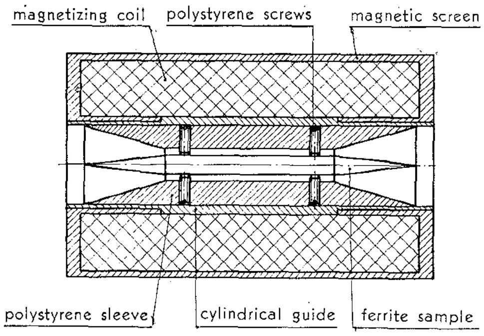


Fig. 2—Schematic cross section through the main part of the IK-1M isolator.

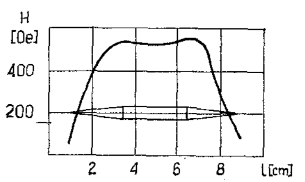


Fig. 3—DC magnetic field distribution on the waveguide axis.

Choice of the ferrite rod diameter for Faraday rotation devices is extremely difficult. Although the figure of merit $F$ against ferrite diameter was determined,[2] this gave little practical information, since $F$ continuously decreases with the increase in the diameter and small diameters are of no use because of great lengths involved.

The following conditions are to be considered in designing ferrite samples for practical Faraday rotation devices:

1) the smallest possible losses of the device,
2) broadband performance (rotation independent of frequency),
3) small dimensions of the device (short ferrite sample—large rotation per unit length; preferably weak magnetic field).

The list conditions may be fulfilled for a given ferrite if proper dimensions (especially the diameter) of the sample are found. The final solution of the problem is in finding a reasonable compromise between the contradictory demands, and the results are, of course, dependent on the kind of ferrite used.

As has been stated before, the figure of merit, generally speaking, decreases with the increase in the ferrite diameter, this effect being caused by a steep increase in the losses, as the specific rotation steadily grows with the diameter (to certain limits). It follows from the above that losses limit the use of large diameters, and the fulfillment of 3) limits the use of small diameters.

In addition to the considerations on the figure of merit, the increase in the ferrite diameter has one more limitation: excitation of higher modes, in particular the hybrid $TM_{11}$ mode (called the $HTM_{11}$ mode throughout this paper), which propagates easily in the structure concerned.

Another problem associated with higher modes propagation in the system is the direct coupling between modes excited by the discontinuities at both ends of the sample. Many spurious effects which have resulted from using short ferrite rods totally vanish after elongating the sample.

The theoretical relation between the rotation and diameter is shown in Fig. 4. The curves were drawn on the basis of calculations made by Waldron.[1] These interesting calculations were not known to the authors at the beginning of this work but the measurements indicated that diameters just below the cutoff value of $HTM_{11}$ should be given special attention. Theoretical confirmation of this conclusion was later found in the paper by Waldron. Optimum ferrite to waveguide diameter ratio was found to be $[d_1/d_0]_{opt} = 0.23$, which is somewhat less than the 0.28 value predicted by Waldron. Samples of this diameter 3 inches long produced 45° rotation at $H = 150$ oersteds with average losses less than 0.3 db (0.2 db was also frequently observed). The tolerance on the diameter is rather sharp: 1 per cent increase causes an easily detectable increase in the losses of about 0.05 db and 1 per cent decrease produced a rapid increase of the magnetic field (approximately 20 per cent).

Faraday rotation is highly frequency-sensitive and measures must be taken to avoid unfavorable effects in devices like isolators or circulators, where the angle of rotation must be kept constant over a substantial frequency band. Various methods of broadbanding were tried in this laboratory and finally the Rowen[3]-Ohm[4] method of dielectric counterbalance was adopted, with some modifications. The effect of reducing the dielectric constant of ferrite by surrounding it by a medium of $\epsilon > 1$ is clearly visible from the curves in Fig. 4. Waldron has observed in his paper that the ferrite di-

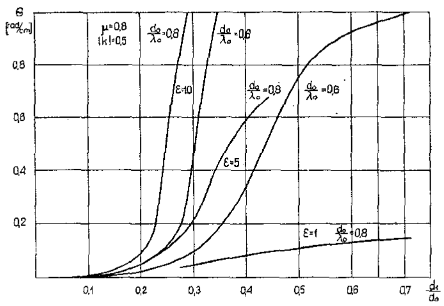


Fig. 4—Faraday rotation per unit length vs ferrite diameter. Influence shown of the surrounding medium and the frequency.

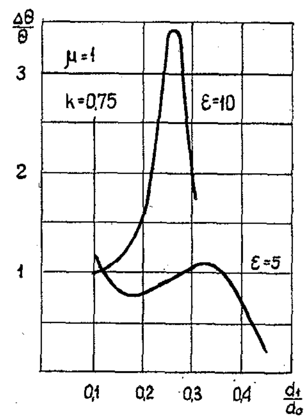


Fig. 5—Relative change in Faraday rotation with frequency ($d_0/\lambda_0 = 0.6 \div 0.8$) vs ferrite diameter.

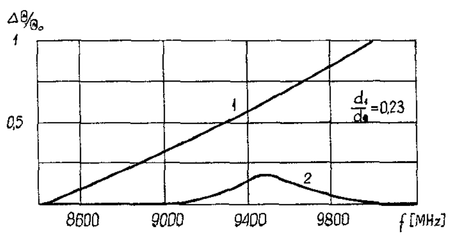


Fig. 6—Relative departures from $\theta = 45°$ as a function of frequency for Ferroxcube B5. Effect of a polystyrene dielectric counterweight demonstrated (curve 2).

[2] S. Lewandowski, "Szerokopasmowy izolator ferrytowy ze strojeniem magnetycznym," *Przeglad Telekomunikacyiny*, Nr. 8–9, 284–288; September, 1958.

[3] J. H. Rowen, "Ferrites in microwave application," *Bell Sys. Tech. J.*, vol. 32, p. 1333; November, 1953.

[4] E. A. Ohm, "A broad-band microwave circulator," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 210–217; October, 1956.

ameter also has an influence on frequency stability of Faraday rotation (Fig. 5). The combining of these two effects proved to give excellent results experimentally. The main advantage of this method lies in the possibility of using easily accessible and low-loss polystyrene instead of high permittivity dielectrics, and in the fact that the ferrite length has no effect on the broad-band performance. Fig. 6 shows the experimental results for Ferroxcube B5, obtained with a 3-inch-long sample of optimum diameter.

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## Equivalence of 0 and −1 Space Harmonics in Helical Antenna Operation*

In considering the propagation of electromagnetic waves along helical conductors using the Tape Helix approximation, it is well known[1] that the solution contains an infinite number of space harmonics. The phase constants of these harmonics are related by

$$\beta_m = \beta_0 + \frac{2\pi m}{p},$$

where $\beta_0$ is the phase constant of the fundamental, $p$ is the helical pitch and $m$ is any integer including zero. It has been shown by Watkins[2] that as far as axial propagation is concerned, it is the −1 space harmonic which is responsible for the operation of the helical antenna. If, however, propagation along the conductor is considered, then the correct space harmonic to be considered is the fundamental as used originally by Sensiper.[3] It is easy to show that both approaches lead to identical results, the proof being as follows.

Let the phase shift between adjacent turns of the helix be denoted by $\theta$ with the subscript 0 or −1; depending on whether the fundamental or the −1 space harmonic is being considered. Then

$$\theta_0 = \frac{L}{\lambda_0}\cdot 2\pi$$

where $L$ is the length of 1 helical turn and $\lambda_0$ is the fundamental wavelength. Denoting the axial velocity of the fundamental by $v_0$, the conductor phase velocity for the fundamental is $v_0/\sin\psi$, where $\psi$ is the helical pitch angle, so that

$$\theta_0 = \frac{Lf}{\left(\dfrac{v_0}{\sin\psi}\right)}\cdot 2\pi$$

$$= \frac{2\pi p f}{v_0}.$$

Similarly,

$$\theta_{-1} = \frac{2\pi p f}{v_{-1}},$$

where $v_{-1}$ is the axial phase velocity of the −1 space harmonic. This is related to the fundamental axial phase velocity $v_0$ by

$$\frac{v_{-1}}{v_0} = \frac{\beta_0 a}{\beta_0 a - \cot\psi}$$

so that $\theta_{-1}$ eventually simplifies to

$$\theta_{-1} = \frac{2\pi p f}{v_0} - 2\pi,$$

which is identical with the expression for $\theta_0$ except for a difference of $2\pi$ which is not significant.

Therefore, it is equally valid to consider either the fundamental or the −1 space harmonic, the first relating to propagation along the conductor, and the second to propagation axially.

As these phase velocities apply to an infinite helix, it is not possible to use them directly for the finite antenna, since it has been found by Kraus[4] that the phase velocity is also a function of the length of the antenna. Nevertheless, it is known[5] that the solution for the infinite case may be used as a means of estimating the bandwidth of the antenna for any pitch angle $\psi$, and it is now shown that both axial and conductor propagation give identical results.

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[1] S. Sensiper, "EM wave propagation on helical structures," PROC. IRE, vol. 43, pp. 149–161; February, 1955.

[2] D. A. Watkins, "Topics in Electromagnetic Theory," John Wiley and Sons, Inc., New York, N. Y.; 1958.

[3] S. Sensiper, "E.M. wave propagation on Helical Conductors," M.I.T. Res. Lab. Tech. Rept. No. 194; 1951.

[4] J. D. Kraus, "Antennas," Mc-Graw-Hill Book Co., Inc., New York, N. Y.; 1950.

[5] T. S. M. Maclean and R. G. Kouyoumjian, "Bandwidth of the Uniform Helical Antenna," presented at URSI Symposium on Electromagnetic Theory, University of Toronto, Toronto, Can.; June, 1959.

## Application of Perturbation Theory to the Calculation of ω-β Characteristics for Periodic Structures*

The effect of small periodic changes in the physical dimensions of closed periodic structures can be investigated using the perturbation theory developed by Müller[1] and later by Slater.[2] From this theory the fractional change in the natural frequency, $\omega$, of a resonant cavity caused by the introduction into the cavity of a small conducting object of volume, $\tau$, is given by

$$\delta\omega/\omega = \frac{1}{2}\,\frac{\int_\tau (\mu_0 H^2 - \epsilon_0 E^2)\,dV}{\int_v \epsilon_0 E^2\,dV}. \qquad (1)$$

The integration in the numerator extends only over the volume of the perturbing object, whereas that in the denominator extends over the entire volume of the cavity, and $E$ and $H$ are the amplitudes of the electric and magnetic fields.

A commonly used technique for determining the $\omega$-$\beta$ characteristic for a closed periodic structure consists of constructing a resonator from an appropriately chosen length of the structure and determining the natural frequencies of the resonator which correspond to the field configurations of interest.[3] If the fields within the unperturbed structure are known, (1) may be used to compute the effect of small changes in the physical dimensions on these natural frequencies. This technique has been used by Vanhuyse[4] in the construction of a linear accelerator using a disk-loaded circular waveguide.

If the perturbations are periodic and if the period of the perturbation is an integral multiple of the fundamental period of the unperturbed structure, the resonant cavity technique may be used to determine the $\omega$-$\beta$ characteristic for the perturbed structure. For this case (1) may be used to relate the $\omega$-$\beta$ characteristic for the perturbed structure to that for the unperturbed structure.

As an illustration, let the initial unperturbed structure be a uniform disk-loaded circular waveguide of radius $b$, and let the perturbed structure comprise cavities alternately of radius $b_-$ and $b_+$ as shown in Fig. 1.

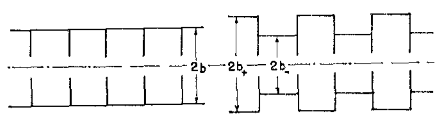


Fig. 1—Uniform and perturbed disk-loaded circular waveguides.

If the average volume per cell is unchanged by the perturbation and if $b_+ - b \ll b$, it is found that the $\omega$-$\beta$ characteristic for the perturbed structure coincides with that for the uniform structure except when the phase shift per section in the unperturbed structure is $\pi/2$. For this situation (which corresponds to a $\pi$ phase shift per section in the perturbed structure), two frequencies are found, indicating the presence of a stop band. The width of the stop band is given by the difference between these two frequencies.

* Received by the PGMTT, November 2, 1959. This work was supported in part by the U. S. Army Signal Engrg. Labs., Fort Monmouth, N. J., under Contract DA 36-039 SC-78254.

[1] J. Muller, "Untersuchung über elektromagnetische hohlraume." *Hochfrequenz. und Elektroak.*, vol. 54, p. 157; November, 1939.

[2] J. C. Slater, "Microwave Electronics," D. Van Nostrand Co., Inc., Princeton, N. J., p. 80; 1950.

[3] B. Epsztein and G. Mourier, "Définition, mesure et caractères des vitesses de phase dans les systèmes á structure périodique" *Ann. Radioélectricité*, vol. 10, p. 64; January, 1955.

[4] V. J. Vanhuyse, "On the proper frequencies of terminated corrugated waveguides with slightly different diameters," *Physica*, vol. 21, p. 603; July, 1955.